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$$= \left\{ \frac{\log n + \frac{1}{n} - \frac{1}{2}\cdot\frac{1}{n^2} + \frac{1}{3}\cdot\frac{1}{n^3} - \dots \text{to } \infty}{\log n} \right\}^p$$

$$= \left\{ 1 + \frac{1}{n\log n} - \frac{1}{2}\cdot\frac{1}{n^2 \log n} + \dots \text{to } \infty \right\}^p$$

$\therefore \lim_{n\to\infty} \frac{u_n}{u_{n+1}} = 1$ and the test fails.

~~Then~~, Now, $\log \frac{u_n}{u_{n+1}} = p \log \left\{ 1 + \left(\frac{1}{n\log n} - \frac{1}{2}\cdot\frac{1}{n^2 \log n} + \dots \text{to } \infty \right) \right\}$

$$= p \left\{ \frac{1}{n\log n} - \frac{1}{2}\left(\frac{1}{n\log n}\right)^2 + \dots \right\}$$

Now,

$$\lim_{n\to\infty} n \log \frac{u_n}{u_{n+1}} = \lim_{n\to\infty} pn \left\{ \frac{1}{n\log n} - \frac{1}{2}\cdot\frac{1}{(n\log n)^2} + \dots \right\}$$

$$= \lim_{n\to\infty} p \left\{ \frac{1}{\log n} - \frac{1}{2}\cdot\frac{1}{n\log n} + \dots \right\}$$

$$= 0 < 1 \text{ [for all values of } p]$$

$\therefore$ The series is divergent for all values of p.

D'Morgan and Bertrand's Test :-

The series Σu_n of positive terms is convergent or divergent according as

$$\lim_{n\to\infty} \left[(\log n) \left\{ n\left(\frac{u_n}{u_{n+1}} - 1\right) - 1 \right\} \right] > 1 \text{ or } < 1$$

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* $= \lim_{n\to\infty} \frac{4n^2+3}{4n^2+4n+1} = \lim_{n\to\infty} \frac{4+\frac{3}{n^2}}{4+\frac{4}{n}+\frac{1}{n^2}} = \frac{4+0}{4+0+0} = \frac{4}{4} = 1$

Now, we apply Raabe's Test fails, De Morgan & Bertrand Test

Alternative to Bertrand's Test

The series $\sum u_n$ is convergent or divergent according as $\lim_{n\to\infty} \left\{\left(n \cdot \log \frac{u_n}{u_{n+1}} - 1\right)\log n\right\} > 1$ or < 1

Ex ① Test the convergency of series

$$\frac{1^2}{2^2} + \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2}x + \frac{1^2 \cdot 3^2 \cdot 5^2}{2^2 \cdot 4^2 \cdot 6^2} \cdot x^2 + \cdots$$

Soln:- Here, $u_n = \frac{1^2 \cdot 3^2 \cdot 5^2 \cdots (2n-1)^2}{2^2 \cdot 4^2 \cdot 6^2 \cdots (2n)^2} \cdot x^{n-1}$

Then, $u_{n+1} = \frac{1^2 \cdot 3^2 \cdot 5^2 \cdots (2n-1)^2 (2n+1)^2}{2^2 \cdot 4^2 \cdot 6^2 \cdots (2n)^2 (2n+2)^2} x^n$

Now,

$$\frac{u_n}{u_{n+1}} = \frac{\cancel{\frac{1^2 \cdot 3^2 \cdot 5^2 \cdots (2n-1)^2}{2^2 \cdot 4^2 \cdot 6^2 \cdots (2n)^2}}}{\cancel{\frac{1^2 \cdot 3^2 \cdot 5^2 \cdots (2n-1)^2 (2n+1)^2}{2^2 \cdot 4^2 \cdot 6^2 \cdots (2n)^2 (2n+2)^2}}} \cdot \frac{x^{n-1}}{x^n}$$

$$= \frac{(2n+2)^2}{(2n+1)^2} \cdot \frac{1}{x}$$

$$\therefore \lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \lim_{n\to\infty} \left(\frac{2n+2}{2n+1}\right)^2 \cdot \frac{1}{x}$$

$$= \lim_{n\to\infty} \left(\frac{2+\frac{2}{n}}{2+\frac{1}{n}}\right)^2 \cdot \frac{1}{x} = \left(\frac{2+0}{2+0}\right)^2 \cdot \frac{1}{x} = \frac{1}{x}$$

Hence, the series is convergent if $\frac{1}{x} > 1$ or $x < 1$

" " " divergent if $\frac{1}{x} < 1$ or $x > 1$

when ~~the~~ $x = 1$ the test fails.

From Raabe's Test

$$\lim_{n\to\infty} n \cdot \left(\frac{u_n}{u_{n+1}} - 1\right) = \lim_{n\to\infty} n \cdot \left\{\frac{(2n+2)^2}{(2n+1)^2} - 1\right\}$$

$$= \lim_{n\to\infty} n \times \left\{\frac{\cancel{4n^2} + 8n + 4 - \cancel{4n^2} - 4n - 1}{(2n+1)^2}\right\}$$